Chapter 4: Integrals
Concepts/Skills to know:

- Find an antiderivative function ***F(x)*** of a function ***f(x)*** by using **antidifferentiation**.
- Know that the derivative of a function's antiderivative is the function itself: ***F'(x) = f(x)*** or $\frac{d}{dx}F(x) = f(x)$
- Identify an **indefinite integral** as a family of antiderivative functions, not any specific function, and evaluate an **indefinite integral**: $\int f(x)dx = F(x) + C$
- Evaluate indefinite integrals containing **integrands** which are any of the following:
  constant functions, linear functions, polynomial functions, rational functions, radical functions, sine and cosine functions, sum of functions, difference of functions
- Solve **differential equations** to find ***f(x)*** using given initial conditions and antidifferentiation.
- Identify the antiderivative of the **acceleration function** as the **velocity function** and the antiderivative of the **velocity function** as the **position function**. Be able to **sketch** and **label** the graphs of these functions.
- Find the indefinite integral of a composite function ***f(g(x))*** by using the substitution method:
  u = g(x) and ***du = g'(x) dx***
- Identify the **definite integral** as area and as a specific number:
  $\int_a^b f(x)dx$ = Area between the graph of ***f(x)*** and the x-axis on [a, b] (***a*** is lower limit, ***b*** is upper limit on x-axis)
- Identify the **area above** the x-axis as **positive** and the **area below** the x-axis as **negative**.
- Find the **Riemann** approximation R_P by finding the sum of the areas of the **rectangles** between the x-axis and the graph of the function by evaluating ***f(x)*** at the left-hand, right-hand or midpoint of each subinterval of **P**.
  n = the number of subintervals (i.e. rectangles).
  The rectangle's top **left**-hand corner needs to be on the graph of the function *or*
  the top **right**-hand corner needs to be on the graph of the function *or*
  the **mid**point of the top (or bottom) needs to be on the graph of the function.
- Evaluate the **definite integral** by regarding it as the **area** between the x-axis and the graph of a function. Given lower and upper limit, find areas of rectangles, triangles, trapezoids, half-circles, and quarter-circles.

Circle Eqn: $(x-a)^2 + (y-b)^2 = r^2$ Center (a,b) radius r top half: $y-b = +\sqrt{r^2-(x-a)^2}$ bottom half: $y-b = -\sqrt{r^2-(x-a)^2}$

$A_{circle} = \pi r^2$ $A_{rectangle} = b \cdot h$ $A_{triangle} = \frac{1}{2}(b \cdot h)$ $A_{trapezoid} = \frac{b_1 + b_2}{2} \cdot h$

- Know that the **integral sign** $\int$ connotes "sum" and the letter **sigma** $\sum$ represents "sum" of numbers.
- Use **properties** of **definite integrals** to evaluate definite integrals:

$$\int_a^a f(x)dx = 0 \qquad \int_a^b c\,dx = c(b-a)$$

$$\int_b^a f(x)dx = -\int_a^b f(x)dx \qquad \int_a^b cf(x)dx = c\int_a^b f(x)dx$$

$$\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx \qquad \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

- Use the **Mean Value Theorem** for Definite Integrals
  to find the average function value f_{av} on [a,b] and find the number ***z*** that satisfies $f(z) = f_{av}$:

$$\int_a^b f(x)dx = f(z)\cdot(b-a) \qquad f_{av} = \frac{\int_a^b f(x)dx}{b-a}$$

- Use the **Fundamental Theorem of Calculus**, Part II, to evaluate definite integrals and get a specific number:

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a) \qquad \int_c^d g(u)du = [G(u)]_c^d = G(d) - G(c)$$